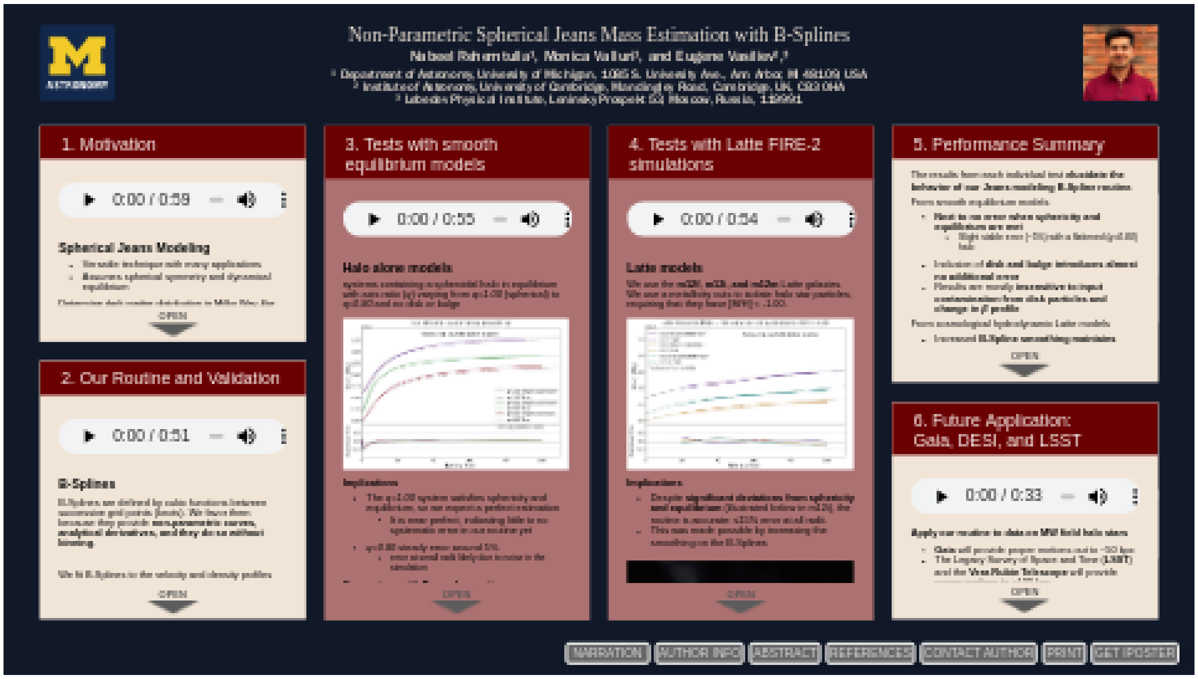
Non-Parametric Spherical Jeans Mass Estimation with B-Splines



Non-Parametric Spherical Jeans Mass Estimation with B-Splines

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237TH MEETING OF THE AMERICAN ASTRONOMICAL SOCIETY
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1. MOTIVATION

Spherical Jeans Modeling

- Versatile technique with many applications
- Assumes spherical symmetry and dynamical equilibrium

Determine dark matter distribution in Milky Way-like galaxies using the estimated cumulative mass profile *M(< r)* from Jeans modeling

The Spherical Jeans Equation

(Jeans 1915, Binney & Tremaine 2008)

$$M(<r) = -\frac{\overline{v_r^2}\, r}{G}\left(\frac{\mathrm{d}\ln\rho}{\mathrm{d}\ln r} + \frac{\mathrm{d}\ln\overline{v_r^2}}{\mathrm{d}\ln r} + 2\beta\right)$$

v_r radial velocity, v_φ azimuthal velocity, v_θ polar velocity, ρ tracer number density, β velocity anisotropy: $\beta = 1 - \frac{\overline{v_\theta^2}+\overline{v_\phi^2}}{2\overline{v_r^2}}$

Most **current implementations of Jeans radially bin data and/or fit power-laws** to velocity and density profiles. (e.g. Xue et al. 2008, Gnedin et al. 2010, Kafle et al. 2018) **Convenient, but with drawbacks:**

- Power-laws are parametric fitting curves
- Binning makes dependence on choice of bins

Goals:

- Develop a novel Jeans modeling routine without these drawbacks and validate it on mock datasets
- Assess effects of non-sphericity and non-equilibrium
- Later, apply our routine to MW halo stars using Gaia, DESI, and later LSST data to **map the outer reaches of our galaxy**

2. OUR ROUTINE AND VALIDATION

B-Splines

B-Splines are defined by cubic polynomial functions between successive grid points (knots). We favor them because they provide **non-parametric curves, analytical derivatives, and they do so without binning**.

We fit B-Splines to the velocity and density profiles

- Penalized spline regression to fit B-Splines to v_r^2, v_φ^2 , v_θ^2
- Penalized spline density estimation for tracer number density ρ

Our fits are performed with Agama (https://github.com/GalacticDynamics-Oxford/Agama) (Vasiliev 2019) as functions of ln(r) and use logarithmically spaced knots.

- Agama reference (https://github.com/GalacticDynamics-Oxford/Agama/blob/master/doc/reference.pdf) section A.2.2 for more information on B-splines

Validating our Routine

We validate the routine with tests on mock datasets of two types:

1. **Smooth, equilibrium** distributions generated with Agama
2. **Latte** (Wetzel et al. 2016): **cosmological hydrodynamic MW-like systems** simulated with the FIRE-2 physics model (Hopkins et al 2018)

Comparison with Power-laws

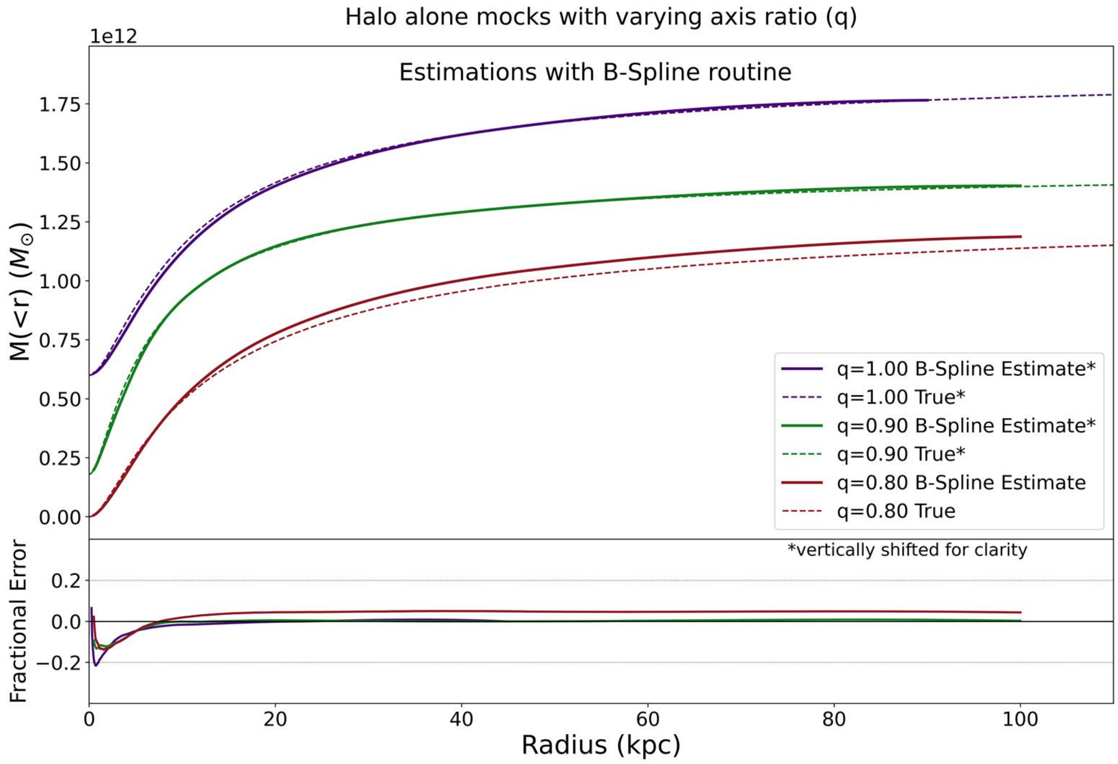
We also implement a routine using power-laws to compare our B-Spline results with.

- Fit power-law of form $y = Bx^A + C$ to v_r^2, v_φ^2 , v_θ^2 , and cumulative tracer count $C(< r)$
    - Calculate ρ with $C(< r) = \int \rho 4\pi r^2 \mathrm{dr}$
- The minimum radius of the estimation must be beyond the break radius of the NFW density profile
    - if not, the density profile doesn't resemble a power-law shape and the estimation is very poor
    - Our B-Spline has no such restriction
- We do not claim this is the best Jeans power-law routine
    - Regardless, it demonstrates how well or how poorly power-laws adapt to different systems

3. TESTS WITH SMOOTH EQUILIBRIUM MODELS

Halo alone models

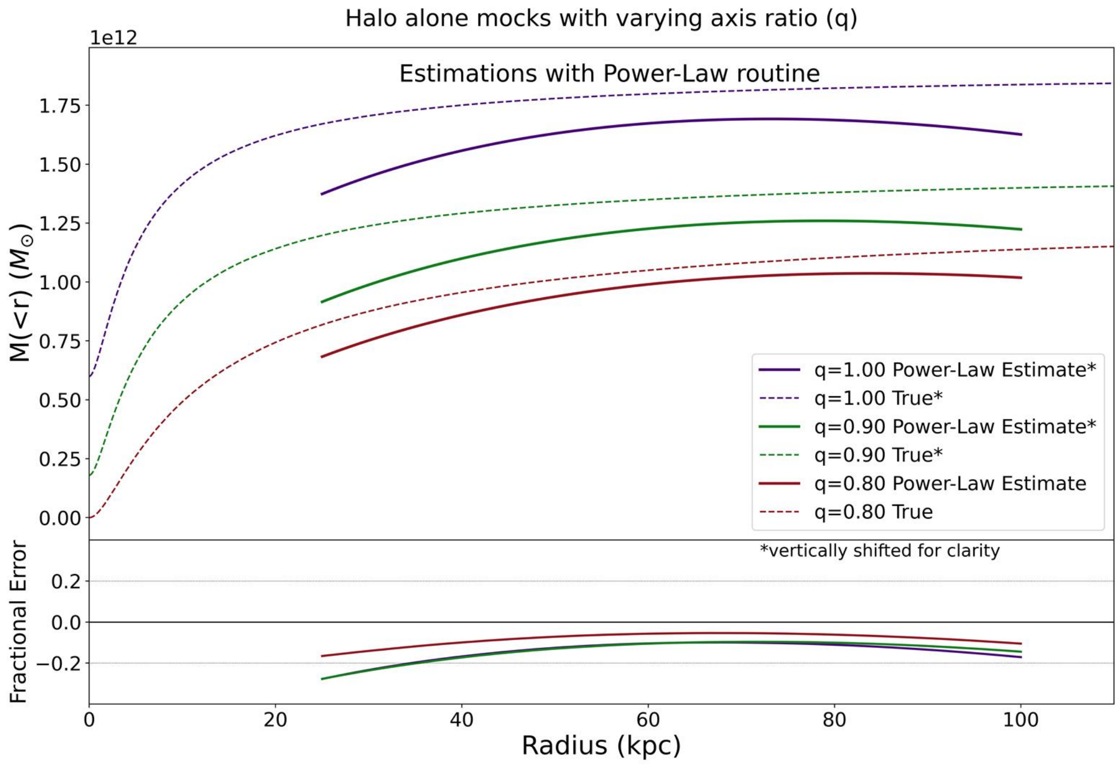
systems containing a spheroidal halo in equilibrium with axis ratio (*q*) varying from *q=1.00* (spherical) to *q=0.80* and no disk or bulge



Implications

- The *q=1.00* system satisfies sphericity and equilibrium, so we expect a perfect estimation
    - It is near perfect, indicating little to no systematic error in our routine yet
- *q=0.80* steady error around 5%
    - error at small radii likely due to noise in the simulation

Comparison with Power-law routine



- Errors greater than B-Splines almost everywhere
- Must evaluate past 25 kpc to avoid the inner density profile shape
  - A single power-law would poorly represent the inner and outer density profiles together

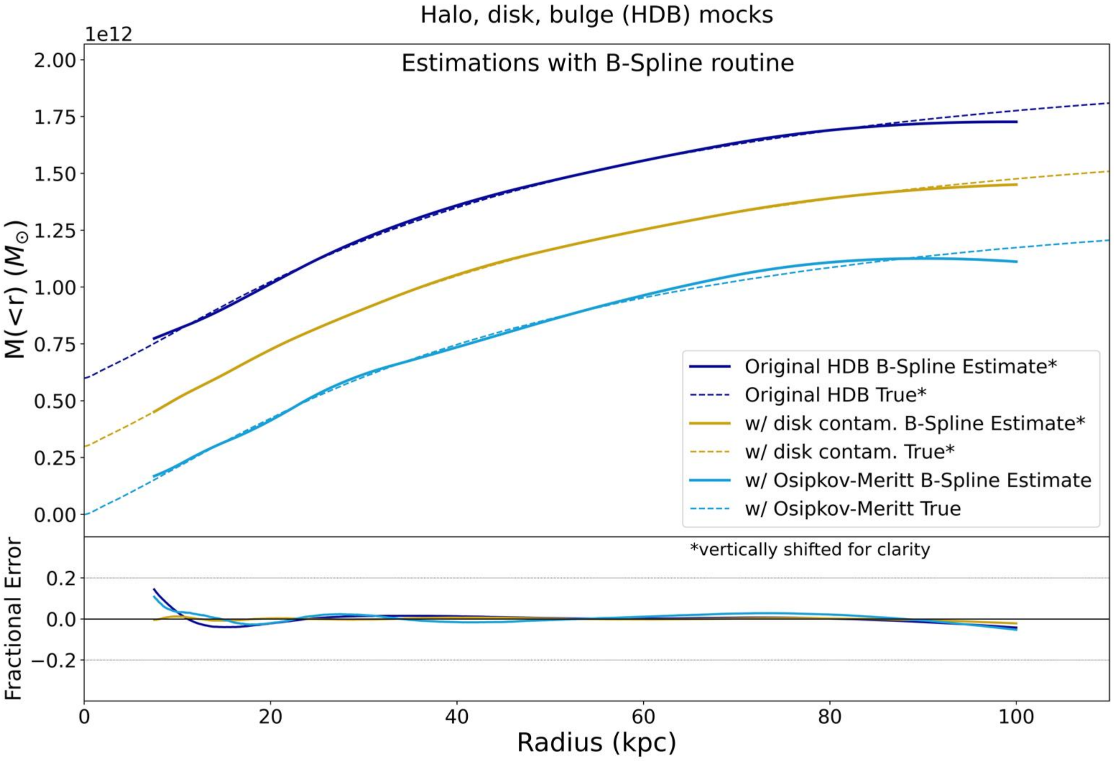
Halo, Disk, Bulge models

systems containing a spherical halo, a disk, and a spherical bulge – all in equilibrium and scaled to be Milky-Way like

- Halo and disk properties from Bland-Hawthorn & Gerhard 2016
  - Disk is a mass-weighted average of the thin and thick disk
- Bulge properties from McMilian 2017

Variations of 'halo, disk, bulge' for additional testing:

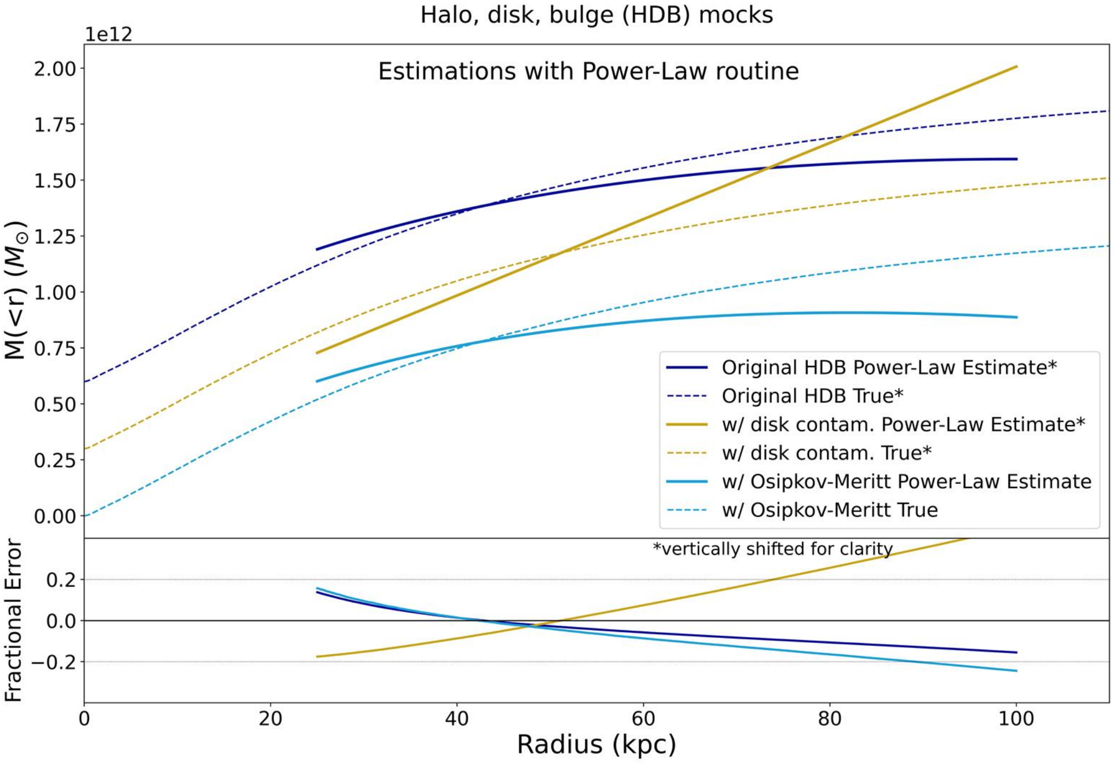
1. Sample 50% of disk particles into Jeans input to test for sensitivity to contamination
2. Recreate the system with a Osipkov-Merritt profile (Osipkov 1979, Merritt 1985a & Merritt 1985b)



Implications

- All three perform excellently with <5% error at most radii
  - The B-Splines adapted well to the different scenarios
- Some error at small radii likely due to noise

Comparison with Power-law routine

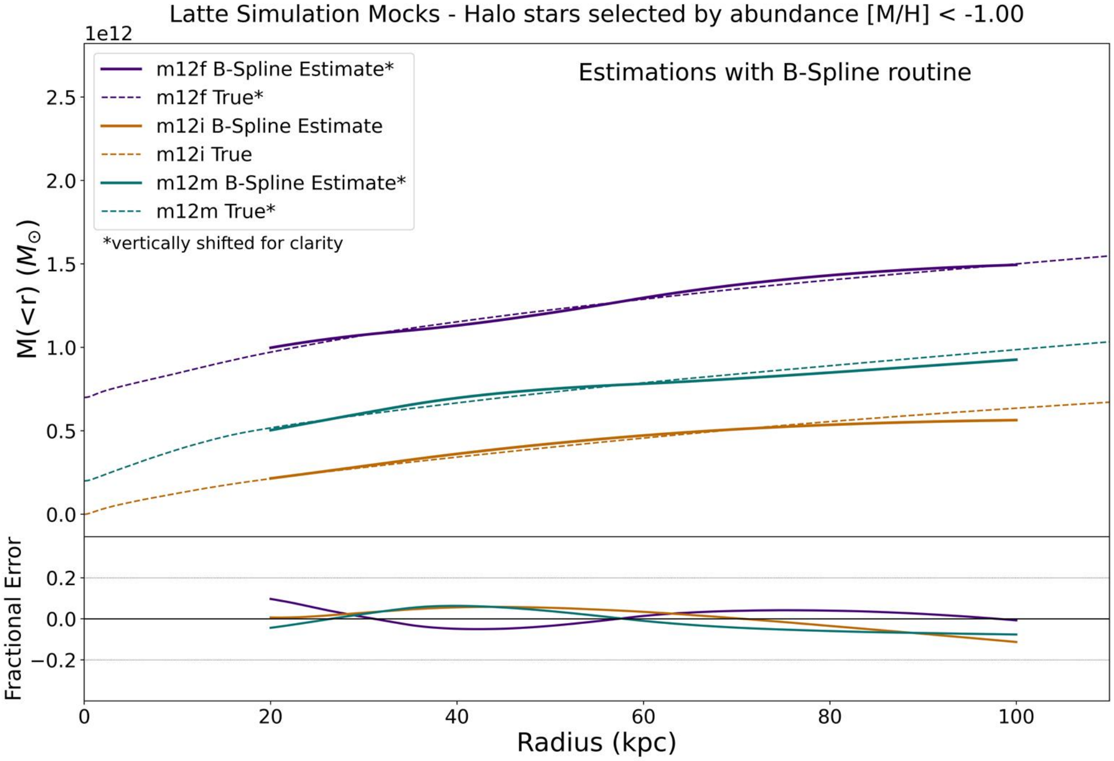


- Larger, less stable errors
- Accuracy is very sensitive to input contamination and slightly sensitive to the Osipkov-Merritt profile

4. TESTS WITH LATTE FIRE-2 SIMULATIONS

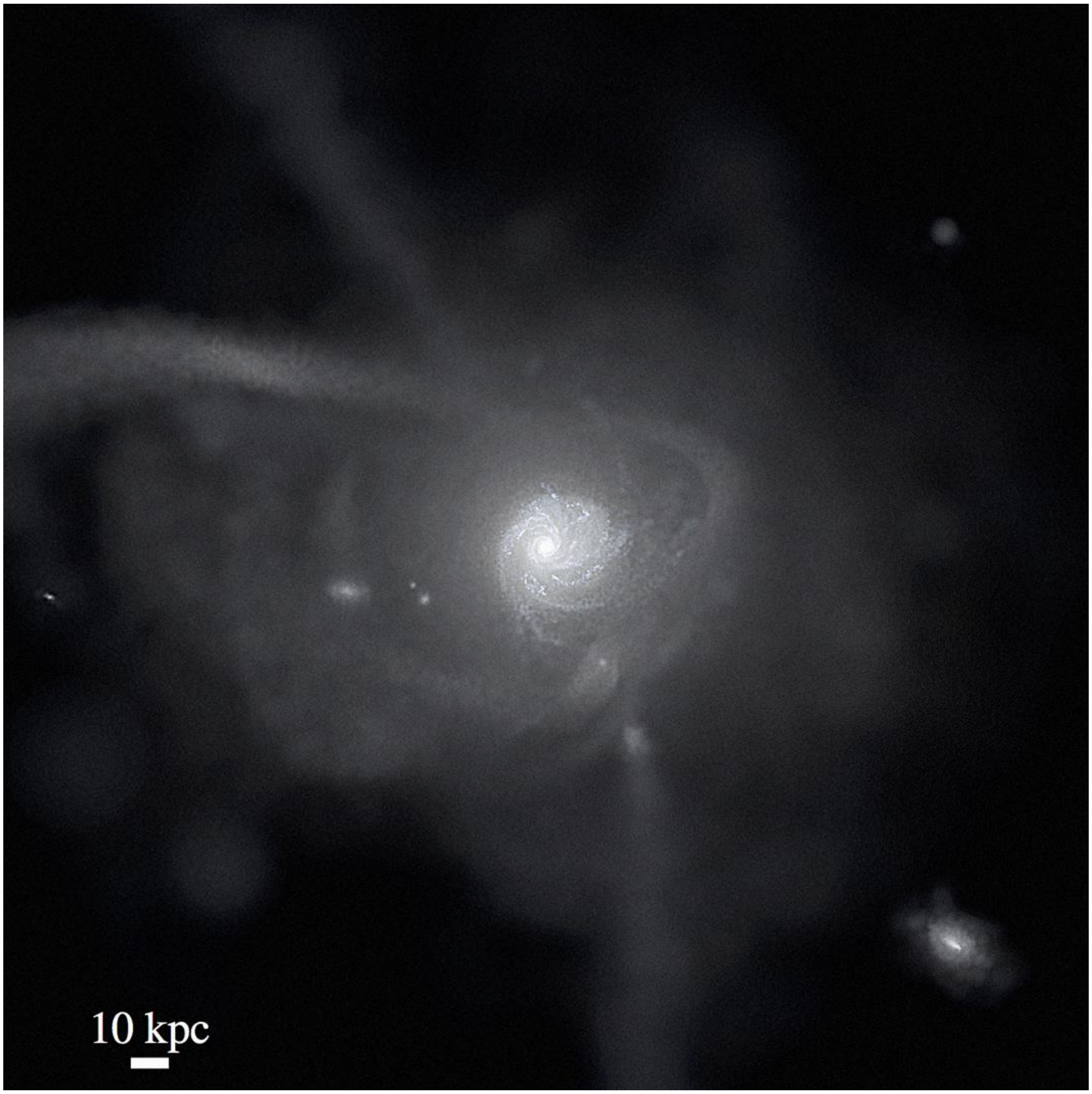
Latte models

We use the **m12f, m12i, and m12m** Latte galaxies. We use a metallicity cuts to isolate halo star particles, requiring that they have [M/H] < -1.00.



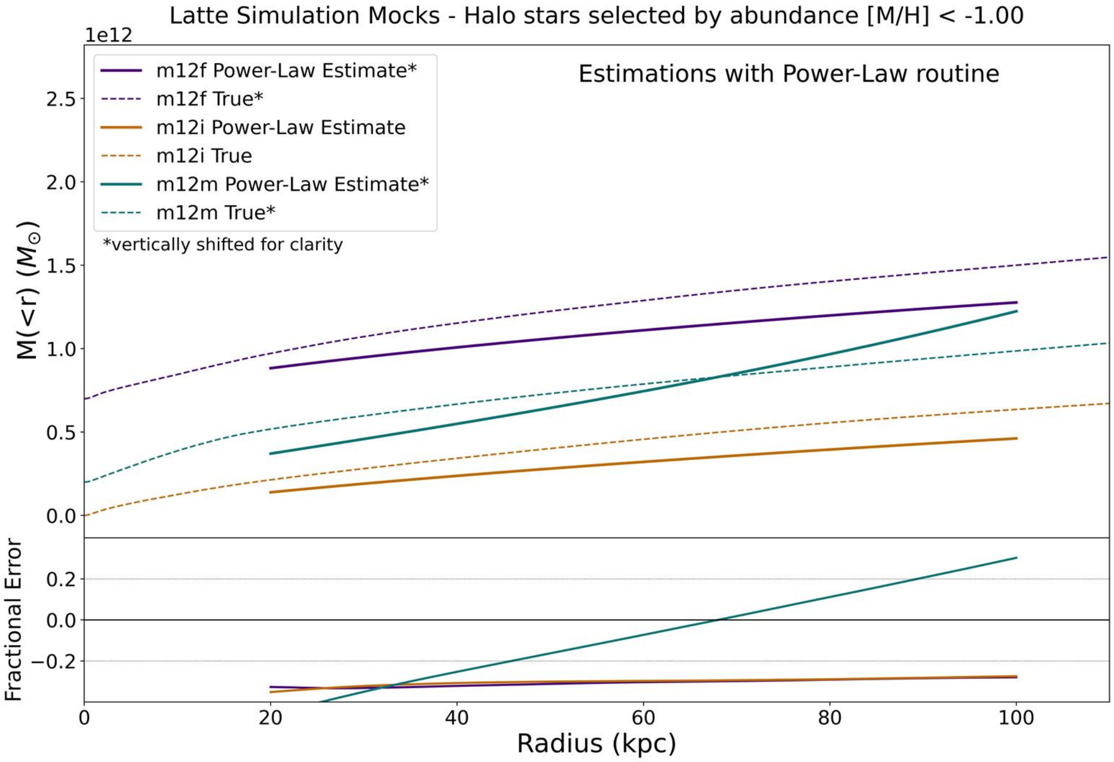
Implications

- Despite **significant deviations from sphericity and equilibrium** (illustrated below in m12i), the routine is accurate: ≤15% error at all radii.
- This was made possible by increasing the smoothing on the B-Splines



Face on image of m12i showing dramatic streams and subhalos (Sanderson et al. 2020)

Comparison with power-law routine



- Much larger errors than B-Splines
- Stability of estimation is model dependent

5. PERFORMANCE SUMMARY

The results from each individual test **elucidate the behavior of our Jeans modeling B-Spline routine**.

From smooth equilibrium models

- **Next to no error when sphericity and equilibrium are met**
  - Slight stable error (~5%) with a flattened ($q=0.80$) halo
- Inclusion of **disk and bulge introduces almost no additional error**
- Results are mostly **insensitive to input contamination from disk particles and change in β profile**

From cosmological hydrodynamic Latte models

- Increased **B-Spline smoothing maintains accuracy** (≤15% error) for systems with very realistic, complex dynamics

From comparison with power-law routine

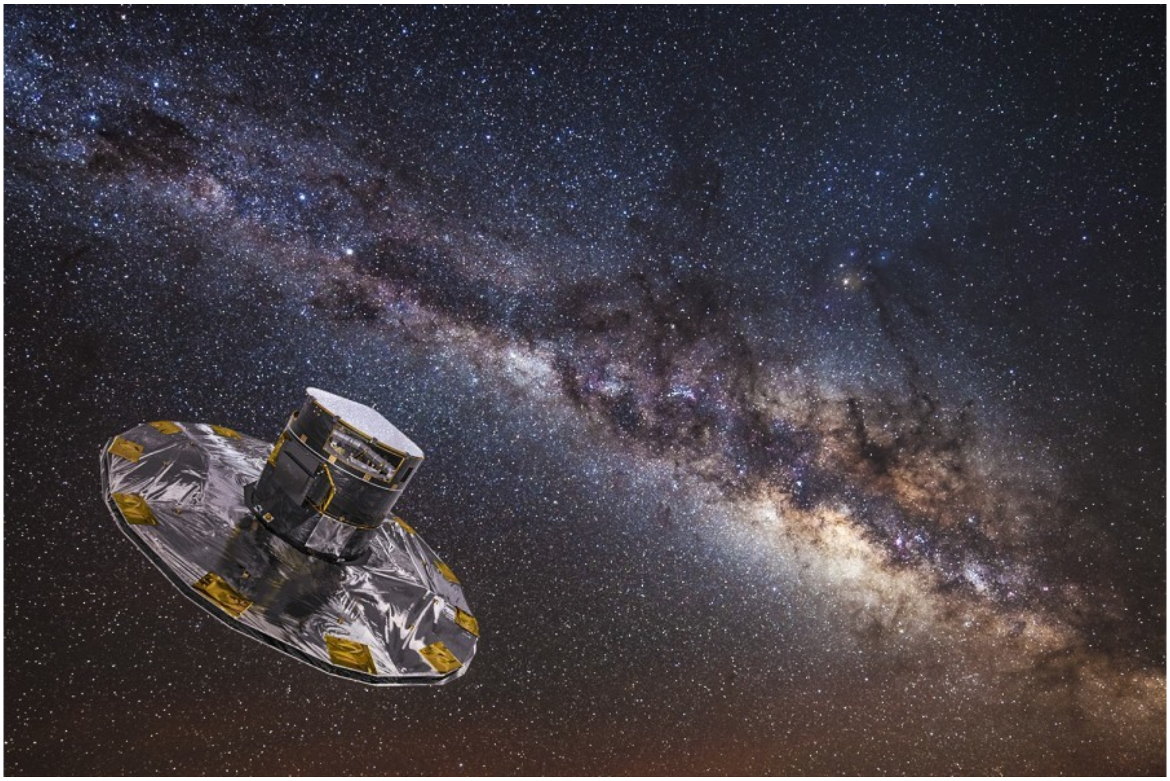
- B-Splines offer **stable, adaptable fits and more predictable errors**
- Error in power-law routine likely comes from the parametric curves poorly representing the velocity profiles

6. FUTURE APPLICATION: GAIA, DESI, AND LSST

Apply our routine to data on MW field halo stars

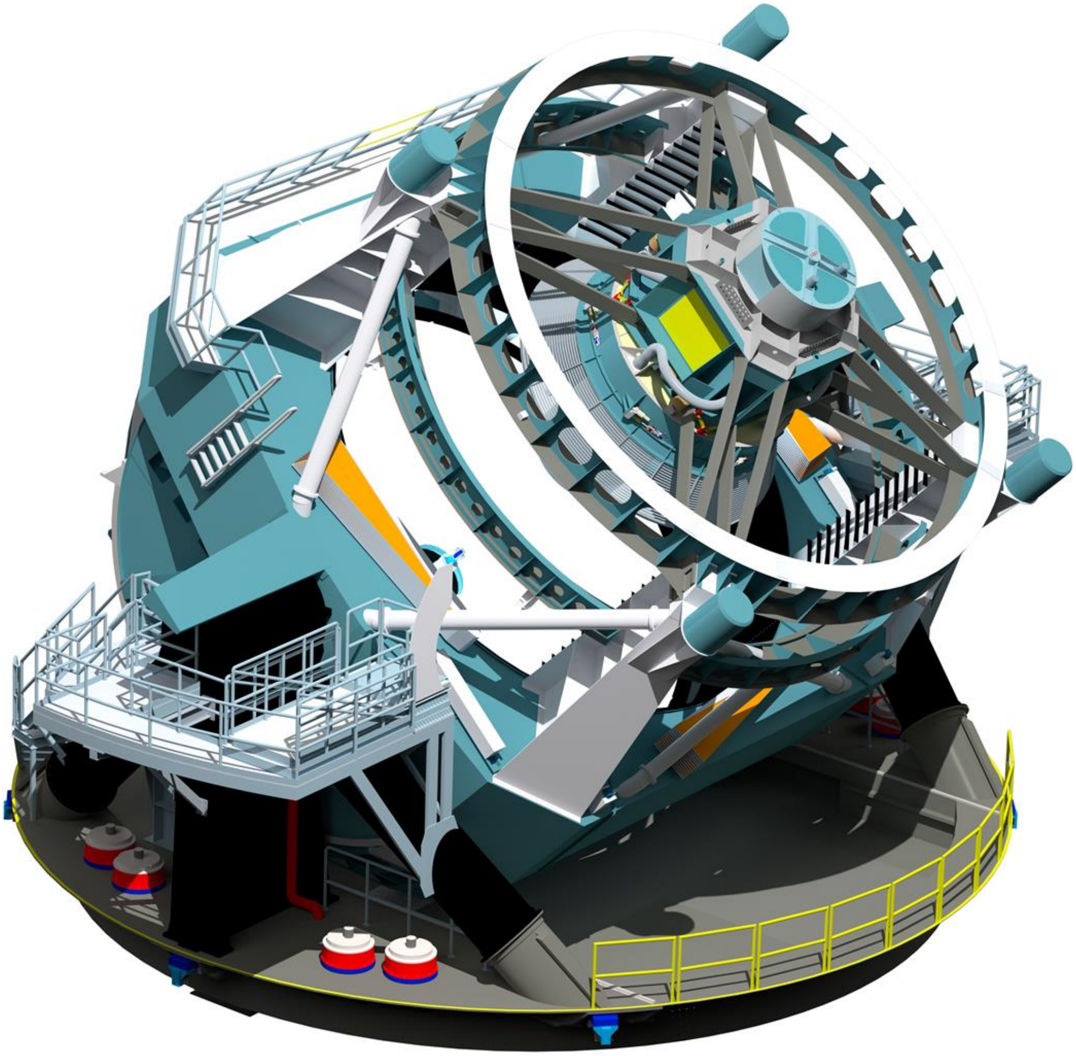
- **Gaia** will provide proper motions out to ~50 kpc
- The Legacy Survey of Space and Time (**LSST**) and the **Vera Rubin Telescope** will provide proper motions to >100 kpc
- The Dark Energy Spectroscopic Instrument (**DESI**) MW survey will provide line-of-sight velocities and spectrophotometric distances to 150 kpc

The result will be an estimated $M(<r)$ profile of the MW **illuminating the distribution of dark matter in our galaxy**.



The Gaia satellite

DESI mounted on the 4-m Mayall telescope



The LSST

AUTHOR INFO

Nabeel Rehemtulla (http://nabeelr.com) is an undergraduate student at the University of Michigan studying astrophysics and computer science. This research is being done as part of his honors thesis and is advised by Prof. Monica Valluri. He is currently applying to graduate programs to pursue a PhD in Astrophysics.

Monica Valluri is a Research Professor at the University of Michigan working in galactic dynamics.

Eugene Vasiliev is a Postdoctoral researcher at the University of Cambridge working in galactic dynamics.

ABSTRACT

Spherical Jeans modeling is widely used to estimate mass profiles of systems from star clusters to galactic stellar halos to clusters of galaxies. It derives the cumulative mass profile, M(<r), from kinematics of tracers of the potential under the assumptions of spherical symmetry and dynamical equilibrium. We consider the application of Jeans modeling to mapping the outer reaches of the Milky Way, specifically to determine the dark matter distribution from field halo stars. We present a novel unbinned non-parametric routine for solving the spherical Jeans equation by fitting B-Splines to the 3-dimensional velocity and density distributions of halo stars obtained by the Gaia survey and spectroscopic surveys such as DESI. While most implementations of Jeans assume parametric forms for these profiles, B-Splines provide non-parametric fit curves and make no sacrifice in the convenience or accuracy of their derivatives. Despite Jeans modeling's prevalence, there is little work quantifying the errors introduced when breaking the assumptions. We validate our routine on several progressively more complex and realistic mock datasets that break these assumptions in different ways. We find that our routine recovers the mass profiles of equilibrium systems with even quite flattened halos and systems including a stellar disk and bulge excellently (<= 5% error). We also perform tests with the non-equilibrium and non-spherical models from the Latte cosmological simulations, which perform reasonably well (<=15% error). This larger error suggests that the output of spherical Jeans modeling is more sensitive to deviations from dynamical equilibrium and the presence of substructure in the halo than deviations from sphericity.

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Software Citations:

Matplotlib (Hunter 2007, Computing in Science and Engineering, 9, 90)

NumPy (Harris et al. 2020, Nature, 585, 357-362)

SciPy (Virtanen et al. 2020, Nature Methods, 17, 261)

Image Sources:

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